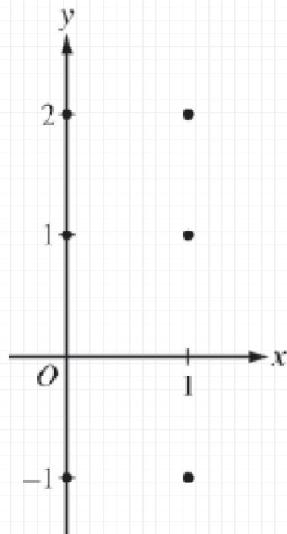


2. A detective is called to the scene of a crime where a dead body has been found. She arrives at 10pm and immediately records the temperature of the body to be $80^{\circ}F$. One hour into her investigation she measures the temperature of the body to be $76^{\circ}F$. She notes that the thermostat is programmed at a constant $68^{\circ}F$. Assuming that the victim's body temperature was normal ($98.6^{\circ}F$) prior to death, when did the death occur?



4. Consider the differential equation $\frac{dy}{dx} = 2x - y$.

- (a) On the axes provided, sketch a slope field for the given differential equation at the six points indicated.



- (b) Find $\frac{d^2y}{dx^2}$ in terms of x and y . Determine the concavity of all solution curves for the given differential equation in Quadrant II. Give a reason for your answer.
- (c) Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(2) = 3$. Does f have a relative minimum, a relative maximum, or neither at $x = 2$? Justify your answer.
- (d) Find the values of the constants m and b for which $y = mx + b$ is a solution to the differential equation.



5. Consider the differential equation $\frac{dy}{dx} = y^2(2x + 2)$. Let $y = f(x)$ be the particular solution to the differential equation with initial condition $f(0) = -1$.

(a) Find $\lim_{x \rightarrow 0} \frac{f(x) + 1}{\sin x}$. Show the work that leads to your answer.

(b) Use Euler's method, starting at $x = 0$ with two steps of equal size, to approximate $f\left(\frac{1}{2}\right)$.

(c) Find $y = f(x)$, the particular solution to the differential equation with initial condition $f(0) = -1$.



x	1	1.1	1.2	1.3	1.4
$f'(x)$	8	10	12	13	14.5

4. The function f is twice differentiable for $x > 0$ with $f(1) = 15$ and $f''(1) = 20$. Values of f' , the derivative of f , are given for selected values of x in the table above.
- Write an equation for the line tangent to the graph of f at $x = 1$. Use this line to approximate $f(1.4)$.
 - Use a midpoint Riemann sum with two subintervals of equal length and values from the table to approximate $\int_1^{1.4} f'(x) dx$. Use the approximation for $\int_1^{1.4} f'(x) dx$ to estimate the value of $f(1.4)$. Show the computations that lead to your answer.
 - Use Euler's method, starting at $x = 1$ with two steps of equal size, to approximate $f(1.4)$. Show the computations that lead to your answer.



$$T - T_s = (\tau_0 - \tau_s) e^{-kt}$$

$$\tau_0 = 80$$

$$\tau_1 = 76$$

$$\tau_s = 68$$

2. A detective is called to the scene of a crime where a dead body has been found. She arrives at 10pm and immediately records the temperature of the body to be 80°F . One hour into her investigation she measures the temperature of the body to be 76°F . She notes that the thermostat is programmed at a constant 68°F . Assuming that the victim's body temperature was normal (98.6°F) prior to death, when did the death occur?

$$\begin{aligned}76 - 68 &= (80 - 68) e^{-kt(1)} & 98.6 - 68 = (80 - 68) e^{-kt} \\8 &= 12 e^{-k} & \frac{30.6}{12} = e^{-kt} \\ \frac{8}{12} &= e^{-k} & \ln\left(\frac{30.6}{12}\right) = -kt \\-\ln(8/12) &= k & \frac{\ln(30.6/12)}{-k} = t \\&& t = 2.30869 \text{ hrs.}\end{aligned}$$

$\approx 2 \text{ hrs } 19 \text{ min } 460$

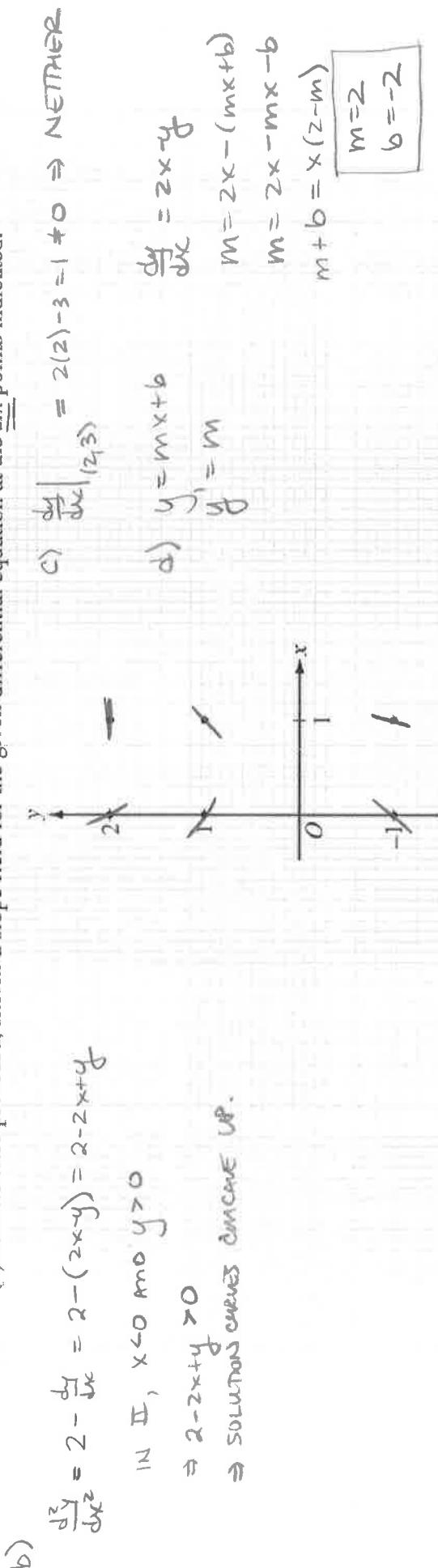
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2015 #4

4. Consider the differential equation $\frac{dy}{dx} = 2x - y$.

(a) On the axes provided, sketch a slope field for the given differential equation at the six points indicated.



(b) Find $\frac{d^2y}{dx^2}$ in terms of x and y . Determine the concavity of all solution curves for the given differential equation in Quadrant II. Give a reason for your answer.

- (c) Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(2) = 3$. Does f have a relative minimum, a relative maximum, or neither at $x = 2$? Justify your answer.
- (d) Find the values of the constants m and b for which $y = mx + b$ is a solution to the differential equation.



2013 # 5

5. Consider the differential equation $\frac{dy}{dx} = y^2(2x+2)$. Let $y = f(x)$ be the particular solution to the differential equation with initial condition $f(0) = -1$.

(a) Find $\lim_{x \rightarrow 0} \frac{f(x)+1}{\sin x}$. Show the work that leads to your answer.

- (b) Use Euler's method, starting at $x = 0$ with two steps of equal size, to approximate $f\left(\frac{1}{2}\right)$.
- (c) Find $y = f(x)$, the particular solution to the differential equation with initial condition $f(0) = -1$.

a) $\lim_{x \rightarrow 0} \frac{f(x)+1}{\sin x} \xrightarrow{\text{L'H}} \lim_{x \rightarrow 0} \frac{f'(x)}{\cos x} = \frac{(-1)^2(2(0)+2)}{\cos(0)} = 2$

b) $\begin{array}{c|cc} x & y \\ \hline 0 & -1 \\ 1/2 & -1/2 \end{array} \quad y_1 = -1 + \left(\frac{1}{1}(2(0)+2)\right)(1/4) = -1/2$

c) $\frac{1}{y^2} dy = 2x+2 dx$
 $-\frac{1}{y^2} = x^2+2x+C$
 $-1 = C$
 $\frac{1}{y} = x^2+2x+1$
 $y = \frac{-1}{x^2+2x+1}$

$f\left(\frac{1}{2}\right) \approx -\frac{11}{32}$

x	1	1.1	1.2	1.3	1.4
$f'(x)$	8	10	12	13	14.5

4. The function f is twice differentiable for $x > 0$ with $f(1) = 15$ and $f''(1) = 20$. Values of f' , the derivative of f , are given for selected values of x in the table above.

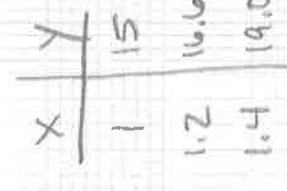
- (a) Write an equation for the line tangent to the graph of f at $x = 1$. Use this line to approximate $f(1.4)$.
- (b) Use a midpoint Riemann sum with two subintervals of equal length and values from the table to approximate $\int_1^{1.4} f'(x) dx$. Use the approximation for $\int_1^{1.4} f'(x) dx$ to estimate the value of $f(1.4)$. Show the computations that lead to your answer.

- (c) Use Euler's method, starting at $x = 1$ with two steps of equal size, to approximate $f(1.4)$. Show the computations that lead to your answer.

a) $y - 15 = 8(x - 1)$

b) $\int_1^{1.4} f'(x) dx \approx 2(10 + 13) = 23(0.2) = 4.6$

$f(1.4) \approx 15 + 8(1.4 - 1) = 15 + 8(0.4) = 15 + 3.2 = 18.2$

c) 

$y_1 = 15 + 8(0.2) = 15 + 1.6 = 16.6$

$y_2 = 16.6 + 8(0.2) = 16.6 + 1.6 = 18.2$

$f(1.4) \approx 18.2$

$19.0 = f(1.4)$